Polynomials

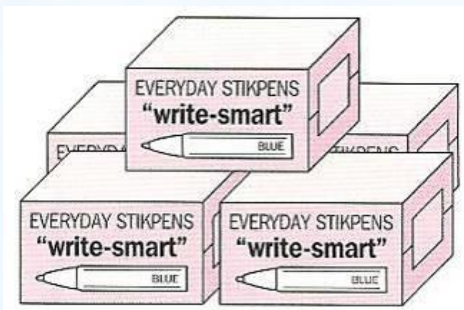
Colegio Herma. Maths. Bilingual Departament by Isabel Martos Martínez. 2015

1. ALGEBRAIC EXPRESSIONS

You can describe everyday situations by using algebra.

In **algebra**, you use letters to represent unknown numbers.

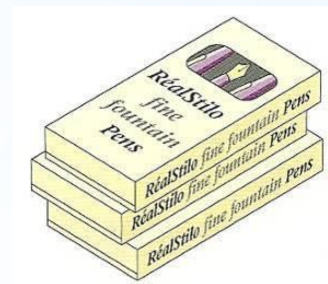
These boxes hold **n** pens each



In 5 boxes there are

$n + n + n + n + n = 5 \times n = 5n$ pens

These boxes hold **s** pens each



In 3 boxes there are

$s + s + s = 3 \times s = 3s$ pens

There are **$5n + 3s$** pens in total.

$5n + 3s$ is an **algebraic expression**.

- An algebraic expression has numbers and letters linked by operations.
- The letters are called variables. Every addend is called term.
- The expression $5n + 3s$ has two terms: **5n** and **3s**.
- You can simplify an algebraic expression by collecting like terms.
- Like terms have exactly the same letters

$3x^2$ and $-5x^2$ are like terms

because they have the same literal part

Example: Simplify these expressions:

a) $4x + 2y - 2x + 3y = 4x - 2x + 2y + 3y =$ **$2x + 5y$**

b) b) $7p - 3q + 5q - p = 7p - p + 5q - 3q =$ **$6p + 2q$**

c) c) $5c - 2b + 2c - 3b = 5c + 2c - 2b - 3b =$ **$7c - 5b$**

Example:

In a fruit shop, apples cost 20p each and oranges cost 15p each. Write an expression for the cost of x apples and y oranges.

Cost of x apples: $20x$

Cost of y oranges: $15y$

Total cost: $\mathbf{20x + 15y}$

Exercise 1

In one month, Dan sends x texts.

a) Alice sends 4 times as many texts as Dan. How many are these?

b) Kris sends 8 more texts than Alice. How many are these?

Exercise 2

In a takeaway pizza

- a medium pizza has 6 slices of tomato
- a large pizza has 10 slices of tomato

How many slices of tomato are needed for c medium pizzas and d large pizzas?

2. MONOMIALS

A **monomial** is the product of a real number (**coefficient or numerical part**) and one or more letters or variables (**literal part**) using only the operation of multiplication and natural exponents.

Monomials have no negative or fractional exponents. ("Mono" implies *one* and the ending "nomial" is Greek for *part*)

The **degree** of the monomial is the sum of the exponents on variables.

Monomial	Coeficient	Literal part	Variable	Degree
$-6x^7$	-6	x^2	x	7
$3x^3 y^2$	3	$x^3 y^2$	x, y	3+2 = 5
$3/5 (a^3 b)$	3/5	$a^2 b$	a,b	3 +1 = 4

Two monomials are **like monomials** when they have the same literal part.

Example:

$2ax^4y^3$; $-3ax^4y^3$; ax^4y^3 ; $5ax^4y^3$ are like monomials

whereas:

axy^3 ; $3a^2x^4y^3$; $2bx^4$ are not like monomials.

Only like monomials can be added together.

Example:

$$5ax^4y^3 - 2ax^4y^3 = 3ax^4y^3$$

$$4ax^4y^3 + ax^4y^3 = 5\ ax^4y^3$$

Opposite monomials are two monomials with the same literal part (like monomials) but opposite coeficients.

Example:

Monomial	Opposite monomial
$-3ax^4y^3$	$3ax^4y^3$
$2bx^4$	$- 2bx^4$
$- 3x^3 y^2$	$3x^3 y^2$

ADDING OR SUBSTRACTING MONOMIALS

Only like monomials can be added together.

Example:

$$5ax^4y^3 - 2ax^4y^3 = 3ax^4y^3$$

$$4ax^4y^3 + ax^4y^3 = 5\ ax^4y^3$$

MULTIPLYING OR DIVIDING MONOMIALS

To multiply monomials we multiply the coefficients of each term and also the literal parts.

Example: $4ax^4y^3 \cdot (-\ x^2y\ ) \cdot\ 3ab^2y^3 = -12a^2b^2x^6y^7$

To divide monomials we divide the coefficients and also the literal parts:

Example: $$\frac{8x^5y}{2x^3} = 4x^2y$$

3. POLYNOMIALS

A **polynomial** is the sum of two or more monomials ("Poly" implies *many*).

- If there are two monomials, it is called a **binomial**, for example

  $x^2 + 2x$

- If there are three monomials, it is called a **trinomial**, for example

  $4x - 6y + 8$

The monomials that form the polynomial are called **terms**.

The **degree** of a polynomial is the degree of the term with the highest degree.

Polynomial	Number of Degree	Classification
5	0	Constant
$3x$	1	Linear or First
$2x^2 + x - 4$	2	Quadratic or Second
$6x^3 - x + 24x^3 - 1$	3	Cubic or Third
$x^4 + 6x^3 - x + 2$	4	Quartic or Fourth

Do exercise 8 and 9 from page 54

- Polynomials are usually written with the terms in **'decreasing'** order

$$2x^4 - x^3 + 3x^2 + 2x + 5$$

that is, with the highest exponent first, the next highest next, and so forth, until you get down to the constant term.

- Polynomials are also sometimes named for their degree:

  - a second-degree polynomial, such as

    $2x^2 - 9$

    $ax^2 + bx + c$, are also called a **'quadratic'**.

  - a third-degree polynomial, such as

    $x^3 - 27$, is also called a **'cubic'**

  - a fourth-degree polynomial, such as

    $2x^4 - 3x + 9$, is sometimes called a**'quartic'.**

- **Evaluating** a polynomial

Is the same as calculating its number value at a given value of the variable.

For instance:

Evaluate $2x^3 - x^2 - 4x + 2$ at $x = -3$

$2(-3\ )^3 - (-3\ )^2 + 4(-3\ ) + 2 =$

$2(\ -27) - 9 + 12 + 2 =$

$-\ 54 - 9 + 12 + 2 =$

$-63 + 14 = -49$

4. OPERATIONS WITH POLYNOMIALS

- To add or subtract two polynomials you add or subtract like terms.

$$\left(4x^2+3x-14\right)-\left(x^3-x^2+7x+1\right)=$$

$$=4x^2+3x-14-x^3+x^2-7x-1$$

$$=-x^3+\left(4x^2+x^2\right)+\left(3x-7x\right)+\left(-14-1\right)$$

$$=-x^3+5x^2-4x-15$$

- To multiply a monomial by a polynomial, you multiply the monomial by each of the polynomial's terms.

For example:

$-3x \, . \, (4x^2 - x + 10) =$

$(-3x)(4x^2) - (-3x)(x) + (-3x)(10) =$

$-12x^2 + 3x - 30x$

Multiply all the terms inside the bracket by the term outside is called **expanding the bracket**.

- To multiply two polynomials you multiply each term in one polynomial by each term in the other, and then add the like monomials (distributive property)

$$\left(4x^2 + 3x - 14\right) \cdot \left(7x + 1\right) =$$

$$4x^2\left(7x + 1\right) + 3x\left(7x + 1\right) - 14\left(7x + 1\right) =$$

$$28x^3 + 4x^2 + 21x^2 + 3x - 98x - 14 =$$

$$28x^3 + 25x^2 - 95x - 14$$

In this case, we **expand double brackets**, that is, each term in the first bracket multiplies each term in the second bracket

- Dividing polynomials

  We can divide polynomials by using two methods:

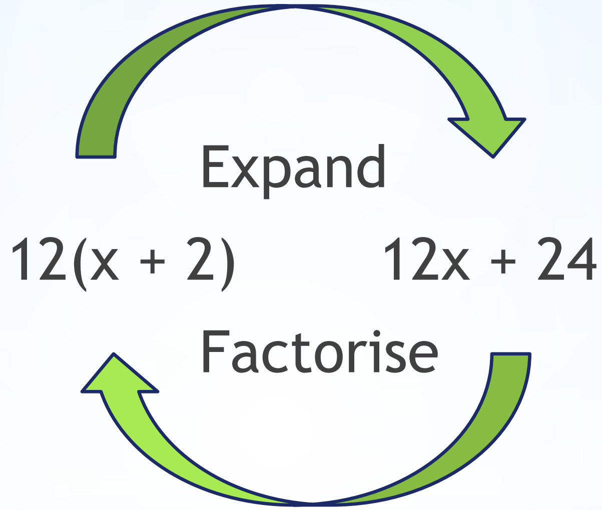
  - The division as if we had natural numbers

  - The division applyin the Ruffini´s method

- **Factorising**

The reverse of expanding a set of brackets is called **factorising**.

To factorise an expression, put brackets in.



To factorise an expression, look for a **common factor** for all the terms.

Examples:

$3x + 9 = 3(x + 3)$

$a^2 - a = a(a - 1)$

$(p + q)^2 - 2(p + q) =(p + q) ((p + q)- 2) = (p + q) (p + q - 2)$

- **Special binomial products (Igualdades notables)**

Some pairs of binomials have ***special products***

When multiplied, these pairs of binomials always follow the same pattern.

By learning to recognize these pairs of binomials, you can use their multiplication patterns to find the product quicker and easier.

Here you are the **SPECIAL PRODUCTS**:

$$(a + b)^2 = a^2 + 2ab + b^2$$

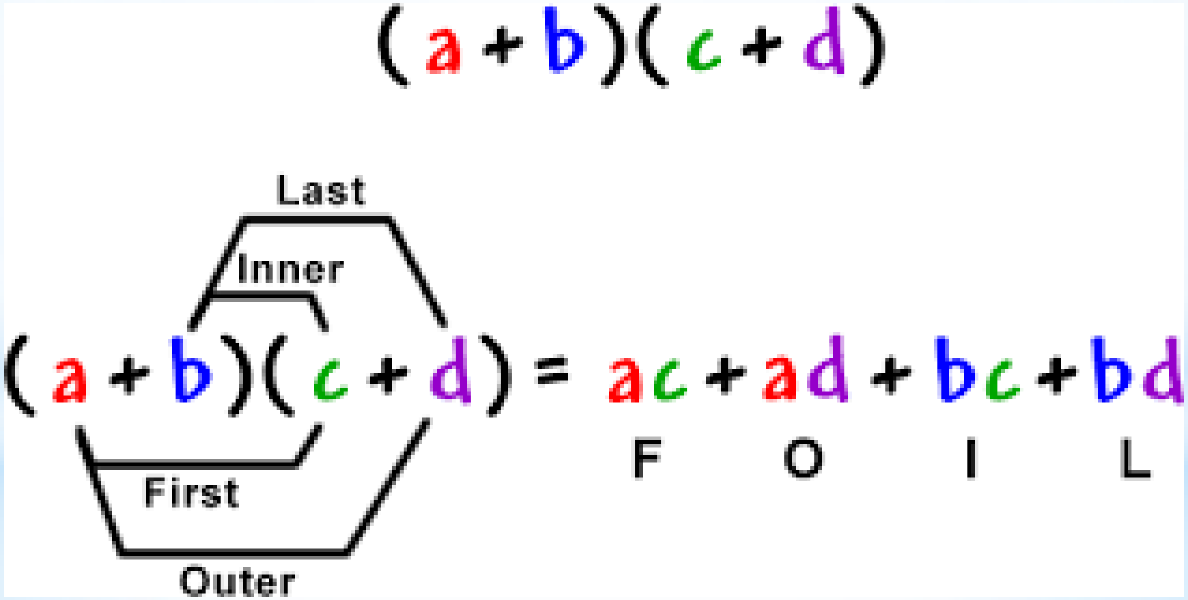
$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

- **Special binomial products (Igualdades notables)**

Multiplication of polynomials is an application of the distributive property. When you multiply two polynomials you distribute each term of one polynomial to each term of the other polynomial.

We can also multiply polynomials by using the FOIL pattern



Some examples:

$(\mathbf{x + 4})^2 = (x + 4)(x + 4) =$

Now we FOIL and collect like terms

$x^2 + 4x + 4x + 4^2 = x + 8x + 16$

In general:

For two numbers *a* and *b*,

$$(a + b)^2 = a^2 + 2ab + b^2$$

Example

$(\mathbf{y - 6})^2 = (y - 6)(y - 6) =$

Now we FOIL and collect like terms

$y^2 - 6y - 6y + 6^2 = y^2 - 12y + 36$

In general:

For any two numbers *a* and b,

$$(a - b)^2 = a^2 - 2ab + b^2$$

Example

$(x + 4)(x - 4) =$

Now we FOIL and collect like terms

$x^2 - 4x + 4x - 16 = x^2 - 16$

In general:

For any two numbers *a* and *b*,

$$(a + b)(a - b) = a^2 - b^2$$

Exercises:

Expand the following expressions:

$(2x + 6)^2$

$(xy + 5x)^2$

$(2x^4 - 8)^2$

$(x^4 - 8)^2$

Find the product of:

$(2x^2 + 9)(2x^2 - 9)$

$(3xy + y^2)(3xy - y^2)$

Write like a binomial product:

$$4x^4 + 20x^2 + 25$$

$$9x^9 - 1$$

$$x^4 - 25$$

$$x^6 - 1$$

Exercise

A small box contains 12 chocolates. Sam buys y small boxes of chocolates.

a) Write an expression for the number of chocolates Sam buys.

A large box contains 20 chocolates. Sam buys 2 more of the large boxes than the small ones.

b) Write an expression for the number of the large boxes of chocolates he buys.

c) Find, in terms of y, the total number of chocolates in the large boxes thatSam buys.

d) Find, in terms of y, the total number of chocolates Sam buys. Give your answer in its simplest form.

Exercise

Jake is n years old.

Jake's sister is 4 years older than Jake.

Jake's mother is 3 times older than his sister.

Jake's father is 4 times older than Jake.

Jake's uncle is 2 years younger than Jake's father.

Jake's grandmother is twice as old as Jake's uncle.

Copy the table and write each person's age in terms of n

Jake	Sister	Mother	Father	Uncle	grandmother
n					